VI.—CRITICAL NOTICES.

The Principles of Natural Knowledge. By A. N. WHITEHEAD, Sc.D., F.R.S. Cambridge University Press. Pp. xii, 200.

THIS book of Prof. Whitehead's seems to me to be very important and distinctly difficult. These facts must be the excuse for the length and the almost wholly expository character of the present review. My main object is, not so much to criticise, as to render what Mr. Bernard Shaw, in the preface to one of his plays, calls 'first aid to critics'. It is a misfortune that the same book should fall twice into the hands of the same reviewer, as has happened in this case. It would be far better to have had the views of two different writers. I can only condole with Prof. Whitehead on his luck, assure him that it was not altogether my fault, and do my best to avoid simply covering the same ground twice over. In a book so rich in matter as this the last task is easier than it would be in many instances.

The book starts with a criticism of the classical concepts of mathematical physics; points, instants, momentary states, unextended particles, etc. It is not denied that such concepts are useful and even indispensable, but the question is: What is their real status? The ordinary physicist rejects such questions as almost indelicate, but for the philosophy of nature it is essential to give some answer to them. The plain straightforward answer is to say that they are particular existents, just as much as anything that we can perceive, and that they are the ultimate constituents of nature. Very few physicists have had the courage to say this and stick to it; the best statement of such a view, so far as I know, is to be found in the last few chapters of Mr. Russell's *Principles of Mathematics.* Even here, however, there is a certain amount of wavering about material, though not about space and time as such. It is insisted that the laws of motion must be expressed in an integrated form as regards time, because a differential coefficient is a mere limit; though for some reason the fact that a density is also a differential coefficient is not seen to lead to the same consequences as regards space and matter. In any case Mr. Russell has long ago deserted this view; and the position of the average physicist seems to be (*a*) that he either says nothing on this delicate subject, or professes himself to believe that the ultimate constituents of nature are extended and that space and time are relative, and (*b*) that, having done this, he always acts as if he believed the opposite. Lastly, when asked what he supposes to be the relation of the sounds and colours which he does perceive to the atoms and molecules which he does not and to the points and instants which are still less like anything perceptible, he either replies that this is 'philosophy' or talks nonsense about sounds and colours being 'unreal'. The idealist philosopher then fastens on these incoherences; informs his readers that physicists move in a world of 'partial appearance' and 'relative truth,' which is quite good enough for persons of their crude understandings; and proceeds to discuss those questions as to whether the Absolute is (or is not) good or happy or a person, which are of such burning interest to minds of finer fibre.

Now the great merit of Whitehead's book I take to be this. He criticises the classical concepts, when taken to be the ultimate existents in nature, as severely as any idealist, though from a far more adequate knowledge and with much less *arrière pensée*. But he also knows that physics cannot get on without them, and believes that the final results of physics are true and verifiable of a large department of nature to a degree to which no philosophical theory can lay the least claim. His problem therefore is this: To define entities which (*a*) shall have the same formal properties and thus do the same mathematical work as the points, instants, etc.; and (*b*) which shall be so connected with the objects that we do perceive and with their perceptible relations that their reality in their own type is as certain as that of the perceptible entities and their relations in *their* type. If he can do this he has killed two birds with one stone. In the first place such entities will no longer be, at best, precarious inferences from what we do perceive (as are atoms or molecules on the usual view), or, at worst, entities which neither resemble what we perceive nor can be inferred from it as hypothetical causes (like points and instants on the absolute theory). They will be instead certain logical functions of what we perceive, defined wholly in terms of it and its relations and of logical constants. Secondly, these entities will now escape the criticisms to which they are exposed when they are regarded as particular existents and the real ultimate existential components of nature. For they now cease to make any such claims, since they are no longer of the type of particular existents but of logically higher types such as classes or classes of classes. They had formerly occupied an embarrassing position in the lowest seat at the feast of nature, and Prof. Whitehead has saved the situation by saying to them: 'Friend, go up higher' (in logical type)!

The object of the book then is to start with the genuine elements of nature which we meet in perception, and their relations; and to exhibit the concepts of physics—modified in accordance with Einstein's first theory of relativity—and *their* relations, as definite logical functions of the former. Thus the work falls into two parts: (i) the determination of the natural elements, and (ii) the detailed exhibition of the concepts as functions of them. In actual fact Prof. Whitehead has accompanied (i) with a general verbal account of (ii), so that it is possible to understand the main drift of the book without reading the detailed logico-mathematical part of it. But a very great part of the value of such a work consists in the detailed proof that the concepts can be connected with the elements, by actually showing the connexion. Other philosophers could have suggested vaguely that the concepts must be some kind of logical function of the elements, but scarcely any except Prof. Whitehead could have worked out the suggestion to a successful conclusion in minute detail. I shall therefore first sketch Prof. Whitehead's view of the elements of nature, and then try to explain the logico-mathematical part of the book.

Nature consists of two fundamentally different but intimately connected types of entity, events and objects. Events are pure particulars, objects are universals. The fundamental connexion between the two is that events are the situations of objects, *i.e.*, an event is characterised by being such and such an object. Events therefore cannot recur in time or space, but objects can, in the sense that different events can be the situations of the same object. Objects are not strictly in space and time and consequently do not strictly have parts. The events which are their situations are in space and time and have parts which are other events. Thus the *event* characterised as 'being a leg of such and such a chair' is a part of the event characterised as 'being such and such a chair'; but the *object* 'being a leg of such and such a chair' is not in the physical sense a part of the object 'being such and such a chair'. It is easy to confuse objects with their situations and thus to imagine that they are in space and have parts.

Events are extended both in space and time. (An event has no special reference to change.) They fall into two great classes, those which are and those which are not durations. An example of a duration is the whole course of nature contemporary with any specious present of any percipient. It is thus limited in time and unlimited in spatial extension. The particular length of anyone's specious present is irrelevant; there are durations of all degrees of temporal extension; the important point is that all have infinite spatial extension and none have no temporal extension. Events other than durations are parts of durations, *i.e.*, are extended over spatio-temporally by durations. This relation of extending over is the fundamental one connecting events. It connects certain pairs of durations, as well as certain pairs of events which are not durations, and durations and the events which are parts of them.

Certain events other than durations have another fundamental relation to a certain duration. They are said to be cogredient with it. This means (*a*) that their temporal extension is the same as that of the duration, and (*b*) that they occupy a fixed spatial position within the duration.

The direct apprehension of events by a percipient consists in his discriminating certain parts of the content of his specious present and regarding them against the undiscriminated remainder. Whitehead apparently holds that the percipient is not only aware in some sense of the undiscriminated background which would ordinarily be admitted to lie in his specious present, but also (though whether in the same sense, I am not sure) of the whole of nature contemporary with this, *i.e.*, with the whole duration.

Events, as we have seen, do not, strictly speaking, change; all that happens to them is that as the course of nature advances fresh durations are juxtaposed on to the front of others. In any duration constituting the content of a specious present the events connected with the mind and the bodily life of the percipient occupy an unique position denoted by the phrase *here-now* in the duration. This event is called the percipient event and it is evidently cogredient with the duration. The ether, according to Whitehead, is the whole continuum of events, and its continuity expresses the facts that any event extends over some and is extended over by other events and that any pair of events are extended over by some third event.

Now there are a great many alternative ways in which a duration can be analysed into events; and the products of different modes of analysis will have different characteristics, *i.e.*, they will be the situations of different types of object. It must not be supposed that there is anything specially subjective or arbitrary about these alternative modes of analysis. We can only analyse out what is actually in nature, and therefore no type of object is more 'real' than another. But some modes of analysis are more useful for one purpose and others for another. The most important modes of analysis lead respectively to events which are situations (*a*) of sense-objects (*e.g.*, sense-data), (*b*) perceptual objects (the chairs and tables, etc., of ordinary life), and (*c*) scientific objects (electrons, etc.). Of these (*a*) are the simplest (*b*) the most useful for everyday life, and (*c*) the most useful for disentangling the laws of nature. But all are equally real in the sense that there really are events in nature which are the situations of objects of each of these types.

Perception is a complicated business. Like all our awareness of objects it implies the power to recognise the same object in different situations (*i.e.*, different events as being instances of the same universal). A perceptual object is an association of sense-objects. Generally we are only aware of a few of these at a time, but they *convey* the rest. Conveyance is not judgment, but is what psychologists term complication and acquired meaning. On this there supervenes a perceptual judgment, part of the contents of which is that the same object (with certain permissible modifications) would be perceived by other percipients from other situations. If this be true the perceptual object is 'real,' otherwise it is 'delusive'. Analysis reveals the fact that objects are only perceived when certain conditions are fulfilled and that the sense-objects which convey the perceptual object vary with these conditions. The conditions split up into two classes, generating conditions and transmitting conditions. When a perception is not delusive the situation of the perceptual object is a generating condition for the sense-object through which the perceptual object is perceived.

The scientific object is the result of further reflexion on the generating conditions of the perception of perceptual objects. The perceptual object is thus a link between sense-objects and scientific objects. Its situation is the situation of the scientific objects which are the generating condition for the sense-objects through which it is perceived. Perceptual objects, though useful for practical life, are not of much use for exhibiting the laws of nature. Their identity and their limits are too vague. Hence we have to replace them for scientific purposes by generating conditions of a more definite kind. The study of these generating conditions leads to the concepts of the atom and the electron; the study of the transmitting conditions leads to the ether, which is not a material object but a continuum of spatio-temporally overlapping events.

An uniform object is one that can characterise an event however short its temporal extension, non-uniform objects can only characterise events of a certain minimum temporal extension. A chair (as perceived), or any other perceptual object, is uniform, a tune or a molecule of iron is non-uniform. Now it might seem that the case of perceptual objects leads to a contradiction. They appear uniform, and they are what they appear. On the other hand they are said 'really to consist' of molecules in motion, and these are non-uniform. The answer is that we must distinguish between the apparent and the causal characteristics of an event. The same event is the situation both of the uniform perceptual object which is the chair and of the non-uniform scientific objects which are the generating causes of the chair being perceived in this situation. Some events are the situations only of causal and not of apparent objects, *e.g.*, events in the ether of space.

If we confined ourselves to sense-objects their laws would be wildly complex, involving as they do generating and transmitting conditions, and, among these, abnormal conditions such as excess of alcohol in the stomach of the percipient. The first step away from these complications is the perceptual object, a complex perceived with slight modification by all normal percipients under all ordinary conditions. We cannot however stop there, partly because of the vagueness of perceptual objects, and partly because we are still left with delusive perceptions on hand. The scientific theory then arises with its scientific objects which are causal in character. Scientific objects are characteristics of an higher order than perceptual objects, they are characteristics of characteristics. Their laws are much simpler than any that we have yet met. Though the presence of a perceptual object in a situation does in fact depend, not only on that situation but also on all other events in the world, yet fortunately it depends predominantly on the scientific objects in that situation, in the case of non-delusive perceptual objects at any rate. Finally, on the basis of what it knows of normal perception, the scientific theory is prepared to deal with the residuum of perception which has delusive objects. It is worth noticing that there is a slight trace of delusiveness in all and a considerable dose of it in some perceptual objects which would usually be reckoned non-delusive. This is because light and sound take some time to travel, so that the situation of the causal components of a given perceptual object is always somewhat earlier than the situation of the perceptual object itself.

From the point of view of science the causal objects seem fundamental and sense-objects mere consequences of them; from that of the theory of knowledge sense-objects seem fundamental and scientific objects mere abstractions from them. The actual truth is that both are equally genuine characteristics of nature, and the differences only rest on the ways in which we get to know them and the use that we make of our knowledge of them.

It is commonly assumed that the ultimate scientific objects must be uniform, in the sense defined above. It is by no means certain that this is true, and in any case non-uniform objects with certain characteristic and recurrent rhythms play a most important part even in pure physics. We can thus see the necessity for some such hierarchy of microscopic and macroscopic equations as Lorentz uses. The electron is uniform; the molecule or atom composed of definite numbers of electrons circulating in definite ways is non-uniform; but once again the collection of many molecules forming a lump of metal is uniform through the averaging out of the rhythms of its component molecules.

Prof. Whitehead suggests, very plausibly I think, that the peculiarity of a living body is that in it we have not a mere average but a macroscopic rhythm. It is obvious that an event characterised as a living being must not be too short; an instantaneous cat is quite as difficult to conceive as Alice found a grin without a cat to be.

I have no space to deal more fully with the philosophical part of the book because I want to try to make the more detailed deductions clear to the reader. To this part then we will now turn.

Events have to each other the fundamental relation of *extending over*, which Whitehead denotes by K. We must remember that an event is best illustrated by a fragment of the content of a specious present. This, in ordinary language, would be said to have some extension both in space and in time. A pair of such fragments may be so related that one spatio-temporally covers the other, and extends beyond it. This is the sort of relation denoted by K. K is an asymmetrical, transitive, relation, and the field of it is assumed to be compact. It is not however connexive, and therefore not serial. This means that, although all events extend over some events and are extended over by others, yet there are pairs of events which do not stand to each other either in the relation K or $\breve{K}$.

The relation K gives us the meaning of *physical* part and whole, as distinct from the merely *logical* part and whole (the relation of a subclass to a class that contains it). The two are often confused, but it is easy to see that they differ when we remember that the physical parts of a whole constitute it by being everywhere adjoined along common boundaries without overlapping. A set of events so related to another event is called a *dissection* of the latter. Whitehead gives logical definitions of dissection, injunction, adjunction, intersection, etc., in terms of K.

One of the axioms laid down for K is that for any two events there is a third event that extends over both of them. This axiom seems to me to be too sweeping and to contradict an important part of the sequel. There is, as we shall see, a certain very important class of events called durations. Durations can only be extended over by other durations. On the electromagnetic theory of relativity (which Whitehead adopts) there are pairs of durations which are not extended over by any third duration (and therefore not by any third *event*). Thus there are events that do not fulfil this axiom, which ought therefore (unless I am talking nonsense) to be restricted to events other than durations.

We next come to the very important concept of an *abstractive class* of events. We have seen that K, when unrestricted, is not serial because it lacks connexity. Now a is an abstractive class if (i) K with its field restricted to members of a is connexive and therefore serial; and (ii) a has no minimum with respect to K. Thus an abstractive class of events is a series of events extending over each other like Chinese boxes and having no smallest box. By means of such classes it is possible to give a meaning to the notion of 'unextended events' without assuming that there literally are such entities in the sense in which there are extended events. This method is called the *Method of Extensive Abstraction*, and, as it is the foundation of the whole building, it is worth while to be quite clear about it. Mathematicians used to define irrationals as the limits of certain series of rationals. The objection to this is that there is no means of proving that such series have limits at all, and therefore irrationals, so defined, may be in the same logical position as the most perfect being or the present king of France. But it was found that the series themselves, whether they have limits or not, have all the properties that irrationals are supposed to have, provided that suitable senses are given to addition, multiplication, etc. And these new senses are such that addition, multiplication, etc., obey precisely the same formal laws as the addition and multiplication in the old sense as applied to rationals. Thus irrationals are *defined* as those series which were formerly said to have irrationals for their limits. The advantages of this procedure are (*a*) that in this sense, there can be no doubt that irrationals exist if rationals do, for these series of rationals are certainly as real as the rationals themselves; and (*b*) that irrationals, so defined, have all the properties that have usually been assigned to them. It is true that, *e.g.*, in the statement $\sqrt{2} \times \sqrt{3} = \sqrt{3} \times \sqrt{2}$ the symbol $\times$ has not the same meaning as in the statement $2 \times 3 = 3 \times 2$. But all the formal properties of the two objects denoted by the now ambiguous symbol $\times$ are exactly the same, and these are the only properties that we make any use of.

Now the Method of Extensive Abstraction is simply the application of the same principle to physics and geometry. We should like to think of points, instants, event-particles, etc., as the limits of abstractive classes. But we have not the least reason to think that such limits exist. On the other hand we cannot get on with our geometry or physics unless we are allowed entities with the properties commonly assigned to points, instants, particles, etc. The solution of the difficulty is found in the fact that the abstractive classes themselves (which as series of events of a certain kind are just as certainly real as the event themselves) or, more accurately, certain functions of them, have to each other relations which possess all the formal properties usually ascribed to the relations of points, instants, etc. We can therefore be sure (*a*) that points, etc., in the sense of certain logical functions of abstractive classes will do all the mathematical work required of such entities, and (*b*) that, in this sense, they are no more fictitious than events themselves, though they are entities of a higher logical type.

Now there are a great many different entities of this abstract kind needed in geometry and physics, *e.g.*, points, lines, planes, instants, instantaneous volumes, momentary point-events, and so on. Thus a great number of special applications of Extensive Abstraction will be needed to define suitable abstractive classes in each case. To set about this work of definition, Whitehead introduces the concept of *primeness* (and *antiprimeness*) of an abstractive class with respect to a *formative condition*. An abstractive class is prime with respect to any formative condition σ when (*a*) it itself possesses the property σ, and (*b*) it is *covered* by any abstractive class that also possesses the property σ. A class β covers a class α if every event in β extends over some event in α. It is thus clear that a class which is prime is a sort of minimum abstractive class out of all those that have a given property σ. Similarly a class that is antiprime is a sort of maximum abstractive class. Antiprimeness is going to lead to moments by way of durations, since a moment refers to a whole of nature spread out in space. Primeness is going to lead to event-particles, *i.e.*, events thought of as unextended in space and time.

So far no restriction has been placed on the formative-condition σ of our abstractive classes. To define moments and particles we must do this. The restriction is that σ shall be *regular* for primes (or antiprimes). σ is regular for primes when (i) there are abstractive classes which are prime with respect to σ, and (ii) all such classes both cover and are covered by each other. (Two classes that fulfil the condition (ii) are said to be *K-equal*. K-equality has the usual properties of equality or identity or equivalence.)

We now define an *absolute antiprime*. This is a class which is antiprime with respect to the condition of covering itself. Such a class covers every class that covers it, and is thus a sort of absolute maximum among abstractive classes. Any member of any absolute antiprime is what we mean by a *duration*. For a duration, as we have seen, is the whole of nature contemporary with the content of a specious present. It is thus an event with a finite temporal and an infinite spatial extension. It is clear that an abstractive class containing events other than durations would not cover *every* class that covered it, since it would be covered by certain classes of durations and would not cover these, because the events in it which were not durations (being of finite extent) could not extend over any duration (since that is of infinite extent). Thus if an abstractive class be an absolute antiprime its members must be durations.

Now this formative condition of covering itself, which is the characteristic mark of abstractive classes of durations, is regular for antiprimes. This means that all the antiprimes that cover any assigned absolute antiprime α are K-equal to each other. In such a case the logical sum of these K-equal classes (*i.e.*, the class whose members are all their members) is called an *abstractive element*. This is defined as the *moment* determined by the abstractive class α of durations. Thus a moment is a certain class of durations, *viz.*, all those durations that belong to any one of a set of abstractive classes which cover an assigned abstractive class of durations.

We are now able to define parallelism of durations and moments, and it is at this point that the question of Newtonian or Lorentz-Einstein relativity enters. If there be a single time-series independent of change of spatial axes, as the classical theory holds, any pair of durations will be extended over by some third duration. But, if Lorentz and Einstein be right and the temporal co-ordinates have to be varied as well as the spatial ones on passing from one set of axes to another in relative motion, it is only the durations of each time series that fulfil this condition; those of two different ones do not. Whitehead adopts the latter view, as indeed we are compelled to do by the facts. He thus gets a definition of parallelism. Durations are parallel when any pair are extended over by a third, otherwise they are not parallel. The moments corresponding to a set of parallel durations are parallel moments. Families of parallel durations and their moments constitute *time-systems*.

I have already said that the supposed existence of non-parallel durations seems to contradict one of the axioms about K. Again we are told that two non-intersecting durations are parallel. I am not clear as to whether this can be proved from the axioms given about K or whether it is to be accepted on the authority of the Theory of Relativity. It is now easy to give a definition of one moment being between two others and thus to establish a continuous serial order among the moments of any time system.

We are then able to define the instantaneous planes, straight lines, and points of a given time system. If we think of thinner and thinner durations within each other we see that they converge to a total state of nature at a moment as an ideal limit, *i.e.*, to an instantaneous three-dimensional 'snapshot' of nature. Now a pair of non-parallel moments intersect. Thus their intersection will correspond to the intersection of two such instantaneous solids, and will be an instantaneous plane in the time-system of either moment. Such an instantaneous plane Whitehead calls a *level*. (For purposes of illustration we have spoken as if there really were these ideal limits, actually they must be replaced, as always, by the abstractive classes and elements which would commonly be said to converge to them. Thus the level l_{12} is really the class of abstractive classes and elements which are covered both by M_1 and by M_2 where these are two non-parallel moments.)

Levels may either be parallel (if *e.g.*, they are the intersections of a moment by two moments of another time system) or they may intersect. Their intersections are called *rects* and are instantaneous straight lines. Lastly two rects may intersect, giving a *punct*, *i.e.*, an instantaneous point in the spaces of the moments in which it lies. The order of puncts on rects in a time-system α depends on the order of the moments in any other time-system β. Every punct on a given rect falls in one moment of β and every moment of β contains one punct on the given rect. And the order will be the same for a given rect whatever other time-system β, non-parallel to its own, we choose to define the order. Puncts, rects, and levels thus form an instantaneous Euclidean space in a moment of a given time-system.

We want now to pass beyond the restriction to single moments in single time-systems, under which we have so far in the main been working. To do this we define an *event-particle*. Event-particles are connected with absolute *primes* in much the same way as moments are connected with absolute *anti*primes. Let κ be any punct. Then an absolute prime connected with κ is an abstractive class fulfilling the following conditions: (i) it must cover every class that belongs to κ and (ii) Any class that fulfils condition (i) must cover it. These conditions (unless I am mistaken) are neatly summed up in the form: $\alpha \epsilon p`\overrightarrow{\Gamma}``\kappa : p`\overrightarrow{\Gamma}``\kappa \subset \overrightarrow{\Gamma}`\alpha$, where α is the class that we are describing, Γ is the relation of covering, and the other symbols have their usual meanings.

It is very easy to prove that the condition just stated is regular for primes; it follows that the logical sum of the class of such classes as α is an abstractive element. This abstractive element is defined as the event-particle connected with the punct κ.

All the event-particles in the whole course of nature form the points of a four-dimensional manifold (Minkowski's 'space-time,' presumably). For a pair of comomental point-events it is clear that the straight line joining them will be correlated with the rect in the momentary space which joins their puncts. But when point-events are not comomental (*i.e.*, are *sequent* in time), it is necessary to give a special definition of lines joining them. This is done in the now familiar way by (*a*) defining linear abstractive classes; (*b*) linear primes; and (*c*), after showing that their formative condition is regular for primes, linear abstractive classes. These are called *routes* and are not of course in general rectilinear. When certain further conditions are imposed on them they become *kinematic routes*, *i.e.*, possible paths for moving material particles. In a similar way *solids* (which may or may not be comomental) are defined and also volumes.

Any finite event can, in a certain sense, be analysed into the set of event-particles that inhere in it. Of course no event-particle is, in the physical sense, a part of an event, since it is an object of an entirely different logical type. (This accords with the common-sense view that, however long you went on dividing up an event or a solid, you would never reach an event that took *no* time or a piece of matter that occupied *no* space.) But there is an unique correlation between any event and a certain bounded set of event-particles which form a continuum; and again, if one event be a *physical* part of another, the set of event-particles correlated with the former will be a *logical* part of the set correlated with the latter. (This accords with the scientific view that extended events and bits of matter can be treated for mathematical purposes as if they were composed of instantaneous states and unextended particles.)

So far we have considered two kinds of manifold, which have characteristic geometries. (i) The three-dimensional Euclidean space of a given instant in a given time-system. (Its points, straight lines and planes are puncts, rects, and levels.) (ii) The four-dimensional 'space-time' whose points are event-particles. So far we have only defined its straight lines in the particular case of comomental event-particles, and we have not defined planes in it. Now neither of these two manifolds is the space of physics. The first is what we approximate to in an observation as the observation takes less and less time; it is thus the sort of thing that psychologists presumably mean when they talk of a perceptual space. The second is neither space nor time but a manifold compounded of both. To complete the geometry of this and to provide the ordinary space of physics whose co-ordinates are the x's, y's, and z's of our differential equations we need a third kind of manifold. This is the space of a given time-system, and may be called a *timeless space* in the sense that, unlike (i), it is neutral as between all the moments of the time-system to which it belongs.

For this purpose we need to make use of the other indefinable relation beside K, *viz.*, *cogredience*, which Whitehead denotes by G. An event is cogredient within a duration when (*a*) any duration of the same time-series that intersects the given duration also intersects the event, and (*b*) the event has an unchanged position within the duration. Practically this means that, if we regard the duration as the content of the specious present of an observer whose perceptive powers were not limited by the spatial remoteness of events from his body, a cogredient event is a part of this content which (*a*) lasts through the whole specious present and (*b*) does not change its position relative to the body of the percipient during the specious present.

By means of abstractive classes of cogredient events we define in the usual way (i) *stationary primes* connected with a given event-particle in a given duration. Then (ii) we prove that the formative condition of such primes is regular for primes and therefore gives rise to an abstractive element. Lastly (iii) we define this abstractive element as the *station* of the given event-particle in the given duration. It will be seen that a station is, roughly speaking, the ideal limit of a set of cogredient events covering the event-particle as these events get thinner and thinner in their spatial extension. A station intersects every moment in its duration in a single event-particle and any one of these particles can equally be taken as the one that determines the station. It can be proved that, if one duration is part of another and P be an event-particle in both, the station of P in the partial duration is a part of the station of P in the total duration. Consequently any station in a duration of a time-system can be prolonged throughout all the durations of that system. The set of event-particles on such a prolonged station is called a *point-track*.

Point-tracks play two parts. They are (*a*) the, as yet undefined, *straight lines* joining pairs of sequent event-particles in the four-dimensional space-time; and (*b*) they are the *points* of the timeless space associated with their own time-system. The straight lines of space-time are now complete except for a certain exceptional kind called *null-tracks* (which correspond, if I am not mistaken, to the generators of the fundamental cone in Minkowski's theory). It remains to define the planes and straight lines of the timeless space of a given time-system, and the planes of space-time.

Just as rects are correlated with some (*viz.*, the comomental) but not all of the straight lines of space-time, so levels are correlated with comomental planes in space-time. But this does not exhaust all the planes in space-time and therefore we need a more general conception, called by Whitehead a *matrix*, which shall include both comomental and non-comomental planes. A matrix is either the comomental event-particles of a level, or is the class of event-particles on all the point-tracks determined by any event-particle in an assigned rect and an assigned event-particle not comomental with that rect. For completeness we must also add the event-particles on the rect through the assigned event-particle which is parallel to the assigned rect. (The reader will observe the analogy of this definition to the definition of a plane in ordinary geometry by a straight line and a point non-collinear with it.)

The elements of the geometry of space-time have now all been defined. It still remains to define the straight lines and planes of the timeless space of a given time-system. A point-track in its own time-system, as we have seen, is a point in the timeless space for that system, for any point-event on it will be in the same station at every moment in the system. The same point-track will intersect the moments of a non-parallel time-system at different stations for each moment in that system. Thus observers in that system will observe a particle moving in a straight line with respect to them. Thus the points of one time-system are the straight lines of any non-parallel time-system. Straight lines in the space of a given time-system can also be defined by means of matrices. If any point-track be chosen the point-tracts which constitute the remaining points of the space of its time-system are said to be *parallel* to it in space-time. A set of parallel point-tracks therefore is a set of points in the space of a single time-system. If the further condition be imposed that the set lies in a single matrix this set constitutes a straight line in the space of the time-system to which they belong.

We may now sum up the information given by Whitehead about the various manifolds that have to be considered in dealing with nature.

MANIFOLD.	POINTS.	STRAIGHT LINES.	PLANES.	PHYSICAL STATUS.
Instantaneous Spaces.	Puncts.	Rects.	Levels.	The ideal limits of perceptual spaces as time is decreased.
Timeless Spaces.	Point-tracks of a parallel family.	Comatricial sets of parallel point-tracks.	?	The spaces contemplated by physics in its differential equations.
Space-time.	Event-particles.	Point-tracks, null-tracks, and sets of co-rect event-particles.	Matrices, and sets of co-level event-particles.	The space-time of Minkowski.

Whitehead does not, unless I have made an oversight, define the planes of a timeless space, but it would of course be easy enough to do this by means of an assigned point-track not on a given matrix and the set of parallel-point tracks on that matrix.

It still remains to set up a system of metrical geometry and c time-measurement for the timeless spaces of time-systems. In order to use rectangular Cartesian co-ordinates it is necessary first to define normality and then to define congruence. The definition of normality is a long and difficult story. It must suffice to say that it is proved that though any point-event sets of three rects which are mutually normal (in a sense defined by Whitehead) exist. Now it will be remembered that a straight line in the time-system of a is a set of parallel point tracks all contained in a matrix of space-time. Any moment of a will intersect this matrix in a rect of the momentary space of a belonging to the given moment; and each punct of this rect will be occupied by an event-particle which belongs to one of the set of parallel point-tracks that constitute the straight line of a-space contained in the matrix in question. Thus there is a correlation between the rect in which a moment of a intersects a matrix associated with a and the straight line of the space of a which is contained in this matrix. The rect is said to *occupy* the straight line. We define mutually rectangular axes in the space of a as the straight lines occupied by the mutually rectangular rects through any event particle in the momentary space of a moment of a. Thus sets of mutually rectangular axes are possible in the space of any time-system.

It may help the reader if I try to indicate the physical meaning of some of these abstract concepts, even though I reverse the logical order in doing so. A point in the space of a would be the position of a particle that stood still as the a-time changed. It will thus appear in space-time as a linear series of event-particles parallel to the t axis, if we choose the time of a as the t axis for space-time. All the other points of a-space will similarly be represented by point-tracks parallel to this t-axis in space-time. Hence the statement that the points of a-space are a family of parallel point-tracks in space-time is explained. A straight line in a-space will represent the successive positions of a material particle as the a-time changes, subject to the condition that these positions are collinear. Each position will be represented in space-time by one point-track, *viz.*, that of a particle which should *permanently* occupy the position in question in a-space. We have seen that all these point-tracks for a given system a will be parallel. It thus becomes clear that a straight line in a-space is represented by a certain selection of parallel point-tracks in space-time. With the same assumption as before about the t-axis for space-time we can regard all the point-tracks which are points in a-space as forming a kind of solid four-dimensional cylinder in space-time with t^a for its axis. A straight line in a-space will then be represented in space-time by the generators of this cylinder which lie on any section of it by a plane containing its axis. Such a plane will be a matrix, it will contain one and only one straight line of a-space and so will be an associated matrix. And it will of course contain other families of parallel point-tracks each of which constitutes a straight line in the space of some other time-system. It is evident that the section of such a matrix by a moment of a will be a rect in a. For this means: Take a set of points in the plane such that t is constant. We shall get a set of point-events that are comomental and collinear, *i.e.*, they will lie on a rect of the instantaneous space of the given moment in a. This will be the rect in the instantaneous space of that moment which is correlated with the straight line of a-space contained in the given matrix.

The definition of congruence is again somewhat difficult. The opposite sides of a parallelogram formed of rects in a level are *defined* as congruent, and stretches on the same rect which are congruent with a third stretch are assumed to be congruent with each other. It is then proved that congruence has this kind of transitiveness even when the two stretches are not on the same rect. So far, however, we have only defined congruence between stretches belonging to rects or point-tracts of parallel families. To extend it to non-parallel families the notion of normality has to be used. If two rects, or a rect and a point-track, intersect at M and are normal, and if AM and BM on one rect or point-track be congruent, then the stretches joining any point on the other rect or point-track to A and to B are defined as congruent. If a certain assumption be made we can show that on any pair of rects congruent pairs of stretches can be found. It is now possible to set up axes for the space of any time-system. If we further assume it to be a law of nature that the velocity of a in the space of β is equal and opposite to that of β in the space of a, when these are any two time systems, we can measure and compare time-lapses. Prof. Whitehead then deduces the connexion between the co-ordinates x_a, y_a, z_a, t_a of an event-particle with respect to the space and time of a and $x_\beta, y_\beta, z_\beta, t_\beta$, the co-ordinates of the same event-particle with respect to the space and time of β. A certain constant κ is involved in these equations of transformation, and according as it is made infinite, negative, or positive we get a Euclidean (Parabolic), elliptic, or hyperbolic type of kinematics. (If it be made equal to O, the results clearly conflict even with quite gross observations.) The elliptic type also conflicts with observation. The parabolic type corresponds with the Newtonian theory of relativity and agrees with observations to a very high degree of approximation. It breaks down, however, in certain very delicate experiments (Michelson-Morley, etc.) whilst the hyperbolic type does not. Thus we are practically tied down to the hyperbolic type, where $\kappa = c^2$ and c is the velocity of light. Whitehead's equations then become identical with those of the Lorentz-Einstein theory of relativity.

It is worth while to note that Whitehead has not needed to make the slightest use of light or its velocity in reaching his transformations. The general form of these has emerged simply and solely from considerations about events, their overlapping, and their cogredience with durations; the definitions of congruence and normality; and the assumption about the velocity of one system in the space of another. It is only at the very last stage, when we ask: What particular value of this general constant κ gives us a system of kinematics that fits all the known facts? that we have to introduce the velocity of light. The existence of such a constant as κ really means that the units in which we measure space and those in which we measure time are congruent with each other.

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